Magic Squares

This problem gives you the chance to:
- work with magic squares, calculating cell values
- understand simple algebraic notation

6	1	8
7	5	3
2	9	4

In this square, adding the numbers in each row, each column and each diagonal gives the same result.

For such a square to be a "magic square" all nine numbers must be different.

1. Find the sum of each row, each column and each diagonal for this magic square. ___________

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x - z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square.

____

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12		14
4		6

4. Find the sum of each row, each column and each diagonal for the completed square. __________

8

Magic Squares		Rubric
The core elements of performance required by this task are: • work with magic squares, calculating cell values • understand simple algebraic notation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **15**	1	1
2. Gives correct answer: **3x**	2	2
3. All five values correct. *Partial credit* 4 or 3 values correct 12 | 1 | 14 11 | 9 | 7 4 | 17 | 6 Some evidence of the correct use of algebra.	2 (1) 2	4
4. Gives correct answer: **27**	1	1
Total Points		**8**

Looking at Student Work on Magic Squares:

Making Sense of Magic Squares Algebraically

There are two parts of the task where students are asked to use algebra. In part 2 of the task, students are to find the sum of the rows and columns for a magic square composed of algebraic expressions. The anticipated response is 3x. About 65% of the 8th graders and 49% of the high school students were able to do this successfully. Some students combined the 3 x's to make xxx or x^3. This shows a basic misunderstanding of how variables are combined and/or the meaning of exponents. Some students forgot the x, or variable, and gave the numerical expression of 3. Some students, about 14% for middle school and 9% for high school, attempted to give a different numerical value for this task.

The second part of the task where students were asked to used algebra was to help solve for the missing values in the magic square, by using the algebraic expressions in the previous example. Students found a variety of solution paths or ways to show their understanding of algebra.

Solution Path 1: Using simultaneous equations

For example:

$$\begin{array}{r} x + y = 12 \\ +\underline{x - z = \;\;4} \\ 2x \quad = 18 \\ x = 9 \end{array}$$

Then using substitution, either mentally or more formally, the student can find the values for y and z. *See work for Student A.*

Student A

6	1	8
7	5	3
2	9	4

In this square, adding the numbers in each row, each column and each diagonal gives the same result.

For such a square to be a "magic square" all nine numbers must be different.

1. Find the sum of each row, each column and each diagonal for this magic square. 15 ✓ 1

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square.

3x ✓ ✓ 2

✓

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

$x+z=12$ $2x=18$ $z=3$ $x=9$
$x-z=6$ $x=9$ $z=3$
$2x=18$ $9+z=12$ $y=5$
$x+y=14$ $9+y=14$
$x-y=4$ $y=5$
$\frac{2x}{2}=\frac{18}{2}$
$x=9$
✓

12	1	14
11	9	7
4	17	6

$9+5-3=11$
$9-5+3=7$
$9-5-3=1$
$9+5+3=17$

2

2

4. Find the sum of each row, each column and each diagonal for the completed square. 27 ✓

Solution Path 2: Using the 3x

In part 2, the students have found that the sum of each row, each column, and each diagonal is equal to 3x. Students could use this total to set up and equation with one unknown and solve for x, which is the center value of the diagonals.

For example:

$12 + x + 6 = 3x$	$14 + x + 4 = 3x$
$18 + x = 3x$	$18 + x = 3x$
$18 = 2x$	$18 = 2x$
$9 = x$	$9 = x$

See work for Student B and Student C.

Student B

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square.

$x+y+x-y+z+x-z$

$3x$

$3x$ ✓ 2 ✓

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

$3x = 18+x$
$-x \quad -x$
$\frac{2x}{2} = \frac{18}{2}$
$x=9$

12	1	14
11	9	7
4	17	6

✓ 2 ✓

$x=9$
$y=5$
$z=3$
$9+y=14$ ✓ 2

$9-z=6$
$-z=3$

4. Find the sum of each row, each column and each diagonal for the completed square. 27

Student C

$x+z$	$x-y-z$	$x+y$	$3x$
$x+y-z$	x	$x-y+z$	$3x$
$x-y$	$x+y+z$	$x-z$	$3x$
$3x$	$3x$	$3x$	$3x$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square.

$3X$ 2

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

$x=9$ $y=5$
$z=3$

$x+z$ 12	1	14	$x+y$
$x+y-z$ 11	9	7	$x-y+z$
4	17	6	$x-z$
$x-y$			

$x+z+x+x-z=x+18$
$3x = x+18$
$-x \quad -x$
$\frac{2x}{2} = \frac{18}{2}$
$x=9$

2

Solution Path 3: Using expressions that cancel out to give one unknown
Looking at the opposite corners in the magic square the algebraic expressions have variables that will cancel out when added. Conveniently the numerical values are also given for these corners.

For example: $(x+z) + (x-z) = 2x$ $\qquad$ $(x+y) + (x-y) = 2x$

$12 + 6 = 18$ $\qquad$ $14 + 4 = 18$

$2x = 18$ $\qquad$ $2x=18$

$x=9$ $\qquad$ $x=9$

See work for Student D. Notice that the student first starts by listing all the givens. This process probably helps the student find the pieces that can be used to set up the equations.

Student D

2	9	4

For such a square to be a "magic square" all nine numbers must be different.

1. Find the sum of each row, each column and each diagonal for this magic square. 15 ✓✓

$6+1+8=15$ $\quad$ $7+5+3=15$ $\quad$ $2+9+4=15$

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. 3x ✓✓ 2

$x+\cancel{z}+x-\cancel{y}-\cancel{z}+x+\cancel{y}$ $\quad$ $x+\cancel{z}+x+x-\cancel{z}$

3x $\quad$ $x+\cancel{y}-\cancel{z}+x+x-\cancel{y}+\cancel{z}$ 3x

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12	1	**14**
11	9	7
4	17	**6**

2

2

$x+z=12$ $\quad$ $x+y=14$

$x-y=4$ $\quad$ $x-z=6$

$x+z+x-z=2x=18$ ✓

$\frac{2x}{2}=\frac{18}{2}$ $\quad$ $x=9$ ✓

$12+9+6=27$

$12+4=16$ $\quad$ $4+6=10$

$27-16=11$ $\quad$ $27-10=17$

$9+17=26$

$27-26=1$

$14+6=20$

$27-20=7$ ✓

4\. Find the sum of each row, each column and each diagonal for the completed ...

Solution Path 4: Substitution
Some students can solve an equation for an unknown and use substitution in second equation to solve for the unknowns.

For example: $x - z = 6$, so $x = 6 + z$

Therefore $(6+z)$ can be substituted for x in the equation $x + z =12$.

So,

$$(6+z) + z =12$$
$$6 + 2z =12$$
$$2z= 6$$
$$z = 3$$

See work for Student E. Notice that all variables are defined after solving for z.

Student E

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. 3x ✓✓ 2

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

x = 9
z = 3 ✓
y = 5

x + z = 12
x - z = 6
x = 6 + z
6 + z + z = 12
2z = 6
z = 3

12	1	**14**
11	9	7
4	17	**6**

✓ ✓ 2 ✓ 2 2

4. Find the sum of each row, each column and each diagonal for the completed square. 27 ✓

Solution Path 5: Four corners

If the algebraic expressions for the four corners are combined, the expression will equal 4x. So an equation with one unknown can be set up to solve for x.

$$(x + z) + (x + y) + (x - y) + (x - z) = 4x$$

$$12 + 14 + 4 + 6 = 36$$

$$\text{so: } 4x = 36,\ x = 9$$

See work for Student F. While the symbolic notation is not present, the thinking and logic are all clearly present.

Student F

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. $\cancel{x^3}$ X

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12 + 14 + 4 + 6 =

36 ÷ 4 = 9 ✓

12	1	**14**
11	9	7
4	17	**6**

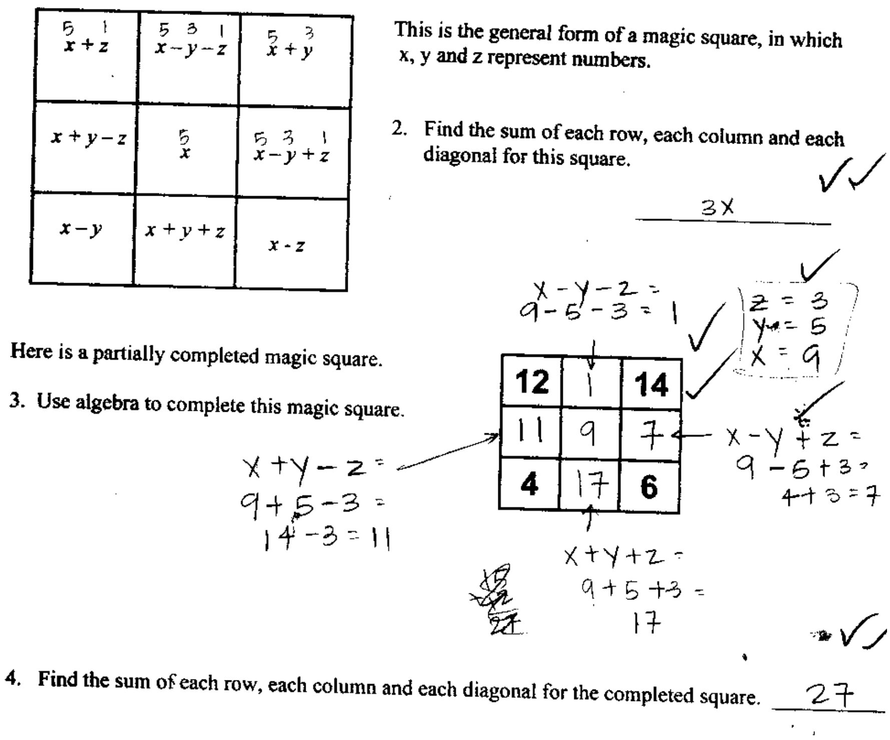
✓ ✓

X 0 X

4. Find the sum of each row, each column and each diagonal for the completed square. 27

Student G uses algebra in the sense of substituting values into expressions with variables. However, there is no evidence to indicate whether the student used algebra or guess and check to arrive at the values for x, y, and z. This is the algebra used by 25% of the eighth graders. No high school students showed just using the value of the variables. Similarly Student H seems to guess to find the total and then uses substitution to solve for the missing values.

Student G



5 1 $x+z$	5 3 1 $x-y-z$	5 3 $x+y$
$x+y-z$	5 x	5 3 1 $x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. 3X

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12	1	14
11	9	7
4	17	6

z = 3
y = 5
x = 9

x − y − z = 9 − 5 − 3 = 1

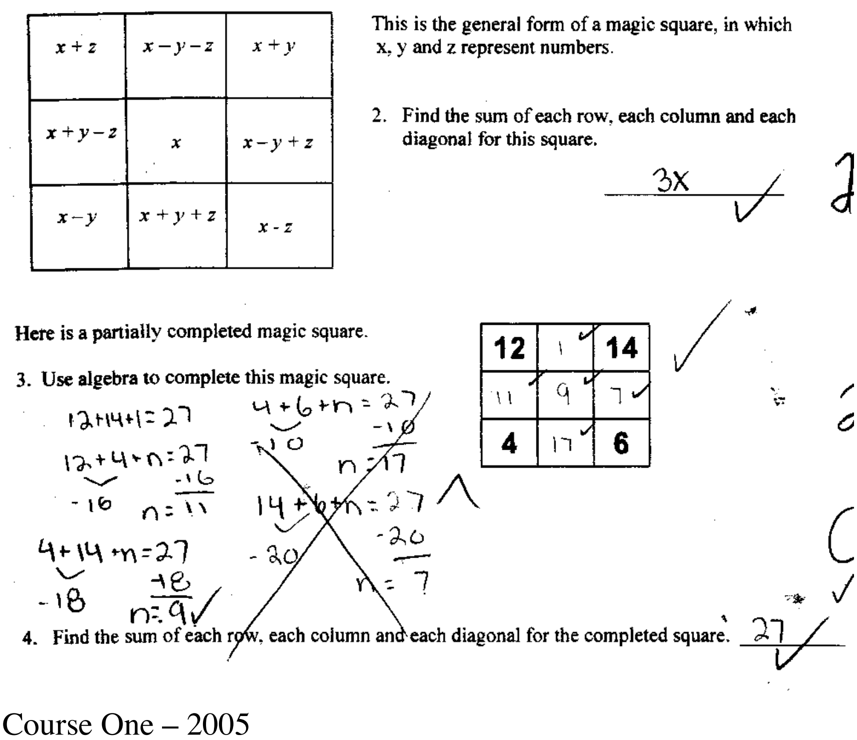
x + y − z = 9 + 5 − 3 = 14 − 3 = 11

x − y + z = 9 − 5 + 3 = 4 + 3 = 7

x + y + z = 9 + 5 + 3 = 17

4. Find the sum of each row, each column and each diagonal for the completed square. 27

Student H



$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. 3X

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12	1	14
11	9	7
4	17	6

12+14+1 = 27

12+4+n = 27
−16 −16
n = 11

4+14+n = 27
−18 −18
n = 9

~~4+6+n = 27~~
~~−10 −10~~
~~n = 17~~

~~14+6+n = 27~~
~~−20 −20~~
~~n = 7~~

4. Find the sum of each row, each column and each diagonal for the completed square. 27

While Student I appears to set up an equation to solve for the unknowns in the magic square. Closer inspection shows that the variables used no longer correspond to the variables in the task. The student then substitutes the different variables, unequal variables, with the same letter designation. So the mathematics of the equations do not lead to the solution of the magic square.

Student I

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. **Find the sum of each row, each column and each diagonal for this square.** 3x ✓

Here is a partially completed magic square.

12	1	**14**
11	9	7
4	17	**6**

3. **Use algebra to complete this magic square.**

12 + 14 + x = 12 + 4 + y
12 + 4 + y

n + 12 + 14 = 12 + n + 4   12 + 14   27   10 17

4. **Find the sum of each row, each column and each diagonal for the completed square.** 27 ✓

Student J is able to combine the algebraic expressions to find the value for the rows equal to 3x. However the student cannot use that information to set up an equation to solve the task. Notice that the student forgets that in this context 3x represents the values of 3 squares in a row, while his numerical expression is the combination of only 2 squares.

Student J

11 + 4 = 15

1. Find the sum of each row, each column and each diagonal for this magic square. 15

$x + z$	$x - y - z$	$x + y$	3x
$x + y - z$	x	$x - y + z$	3x
$x - y$	$x + y + z$	$x - z$	3x

$x + z + (x - y - z)(x + y)$

$3x +$

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square.

3x

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

	x	x
12		**14**
4		**6**

28 21

$\cancel{3x = 12 + 14}$

$\cancel{3x = 26}$

$x = 8$

$x - y + (x + y + z)(x - z) = 3x$

4. Find the sum of each row, each column and each diagonal for the completed square. X

Student K also attempts to set up equations which should be equal to 3x. The student adds more variables to the problem, which further complicates the task of solving the equations because there is no known relationship between n or k and the variables in the original task. The student seems to have used guess and check with an incorrect guess at the value of the totals for the rows and columns. While the student's solution is correct for all the rows and columns, it does not work for the diagonals.

Student K

1. **Find the sum of each row, each column and each diagonal for this magic square.** 15 ✓

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. **Find the sum of each row, each column and each diagonal for this square.**

$x+z-x+x-z = 3x$ $3x$ ✓

Here is a partially completed magic square.

3. **Use algebra to complete this magic square.** X X

$12+x+4=a$
$14+n+6=a$
$4+k+6=a$
$12+y+14=a$
$12+z+6=a$
$14+z+4=a$

$z=10$
$x=12$
$k=8$
$n=8$
$y=2$

$12+10+6=28$
$12+12+4=28$
$4+18+6=28$
$14+8+6=28$
$12+2+14=28$

12	y 2	**14**
x 12	z 10	n 8
4	k 8	**6**

X X

4. **Find the sum of each row, each column and each diagonal for the completed square.** 28

Student L does not know how to add variables with negative values. The student also attempts to use guess and check. The solution works for most rows and columns, but not all. Again, the student does not seem to consider the value of the diagonals.

Student L

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

This is the general form of a magic square, in which x, y and z represent numbers.

2. **Find the sum of each row, each column and each diagonal for this square.** $3x+2y+2z$ X X

Here is a partially completed magic square.

3. **Use algebra to complete this magic square.** X X

12	10	**14**
10	8	6
4	16	**6**

X X

4. **Find the sum of each row, each column and each diagonal for the completed square.** 26 X X

Student M does not understand combining like terms and probably does not understand the meaning of exponents. The student's solution to the magic square is good only for the rows, but not the columns or the diagonals.

Student M

$x+z$	$x-y-z$	$x+y$
$x+y-z$	x	$x-y+z$
$x-y$	$x+y+z$	$x-z$

x^2y

This is the general form of a magic square, in which x, y and z represent numbers.

2. Find the sum of each row, each column and each diagonal for this square. 0

x^2y

0

0

Here is a partially completed magic square.

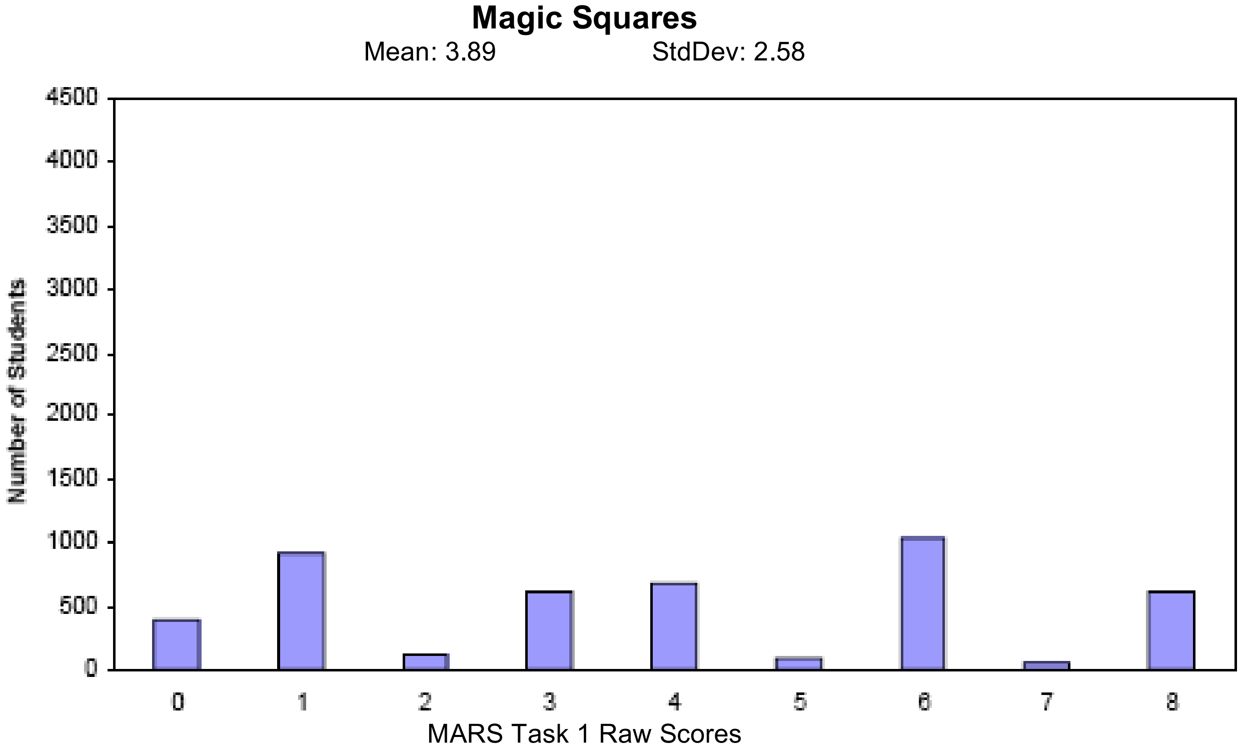
3. Use algebra to complete this magic square.

12	0	**14**
8	8	10
4	16	**6**

4. Find the sum of each row, each column and each diagonal for the completed square. 26

Teacher Notes:

Frequency Distribution for Task 1 – Course 1 – Magic Squares



Score:	0	1	2	3	4	5	6	7	8
Student Count	391	925	124	618	686	77	1037	48	606
% < =	8.7%	29.2%	31.9%	45.6%	60.8%	62.5%	85.5%	86.6%	100.0%
% > =	100.0%	91.3%	70.8%	68.1%	54.4%	39.2%	37.5%	14.5%	13.4%

The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, about 91%, total the numerical values of the rows and columns in the first magic square. Many students, 68%, could total the values of rows and columns in the magic squares both numerically and using symbolic notation. About half the students, could add the number values for the first magic square, fill in numerical values for the third magic square, and give the total for the rows and columns in the third magic square. Some students, 39%, could find the totals for all the magic squares using numbers and symbolic expressions and fill in most of the missing values in the third magic square. 13% of the students could meet all the demands of the task including using algebra to find the missing values for the last magic square. Almost 9% of the students scored no points on this task. 76% of those students attempted the task.

Magic Squares

Points	Understandings	Misunderstandings
0	76% of the students with this score attempted the task. All the middle school students attempted the task.	Most middle school students who missed part one added all the rows, columns, and diagonals to get 120 instead of 14. There was no pattern for errors in high school responses to part 1. Their errors ranged from 3x, f, 34, up to 2098.
1	Students could find the numerical sum of rows and columns for the first magic square.	Students could not find the correct sum for the algebraic expressions in the second magic square. 19.5% did not attempt part 2. 12% repeated the numerical sum from part 1. 19.5% gave a different numerical response. <u>Almost 10% gave the answer of x.</u> *6.5% gave the answer x^3 instead of 3x. 7% gave a different expression using exponents.*
3	Students could find numeric sums and algebraic sums for rows and columns in magic squares.	They had difficulty finding the sum of the third magic square, where some values were missing. 44% of the students did not attempt this part. 11% gave responses of 30, 11% gave responses of 28. 8% gave responses of 26. These responses were usually correct for rows and columns, but incorrect for the diagonals.
4	Students could find the numeric sums for the first square, fill in the missing values in the third square and give its numeric sum.	Students could still not combine like terms in part 2 and did not use algebra to find the missing values in part 3.
5	Students could combine like terms, find the numeric sums of magic squares, and fill in most of the values in the final magic square.	Students made one or two errors in filling in the final magic square and did not use algebra to find the values in the magic square. 34% of all students showed no work for how they solved part 3. 23% of middle school students showed their addition calculations, only 6% of high school students showed addition calculations.
6	Students could add rows and columns numerically and with symbolic expressions and fill in values to make a magic square.	Students did not use or show the use of algebra to solve for the missing values in part 3.
8	Students could add rows and columns numerically and symbolically to find the totals for magic squares. Students could use algebra to find the missing values. <u>25% of all middle school students could show the values of x,y, and z and substitute those values into the expressions in the second square. 10% set up equations or systems of equations and solved for unknowns.</u>	

Underlined statements indicate an error patterns found more exclusively in middle school. Italic statements indicate an error pattern found more exclusively in high school papers.

Based on teacher observation, this is what algebra students know and are able to do:

- Add numerical data
- Use guess and check to find missing values in a magic square

Areas of difficulty for algebra students:

- Combining like terms
- Understanding the difference between $x + x + x$ and x^3
- Using algebra to find a solution in a problem-solving setting
- Setting up an equation or system of equations to match a problem situation
- Using as few variables as possible/ trying to define one term in terms related to other ideas or parts of the problem instead of inserting new variables
- Understanding that the sums of the diagonals needed to be the same as the sums for the rows and columns in a magic square

Strategies used by successful students:

- Finding the value of x, y, and z, then using substitution to find missing variable
- Setting up simultaneous equations with 2 unknowns
- Looking for expressions with similar terms with one value that would cancel out
- Using the diagonal total to set up equation with 1 unknown
- Comparing symbolic and numeric values for the 4 corners

Questions for Reflection on Magic Squares:

- Could your students combine like terms to find the sum of the rows in columns in the second magic square? Are any of your students confused about exponents?
- Did your students show work for part 3, whether it was for guess and check, addition, or using algebra?
- What are the norms in your class for showing work? How does the work help inform you in making lesson plans throughout the year? How do you find out misunderstandings and make adjustments in your lessons?
- Are there some types of assignments where students are given partial credit for correct thinking that could have led to correct answers
- Were students in your class willing to attempt problems they were unsure of? How do you help students develop confidence in their abilities to tackle new types of problems and reason about mathematics using their prior knowledge?
- How often do students in your class have the opportunity to develop their own equations from word problem or problem-solving situations? Do they know to try and define ideas in terms of other information in the problem or do they use a new variable for every idea? Why is it important to use the smallest number of variables? Have students had opportunities to confront and discuss this idea?

Look at student work in part 3. How many of your students:

Showed addition to solve problem, guess and check, totals	
Set up two equations with two unknowns.	
Used the row total, 3x, to set up an equation with one unknown.	
Used substitution or expressions that would cancel out one of the terms.	
Set up equations that didn't match the situation.	
Set up equations, but could not do the algebra to solve the equations.	
Found (method unknown) the values of x,y, an z and showed the substitution of these values to find the missing numbers in the magic square.	
Showed no work.	
Found values that worked for rows only or rows and columns but not the diagonals.	
Did not attempt this part of the task.	

- What does this make you think about it terms of what your students are learning about algebra and their ability to use it in context?
- Do you think your students could have solved the problems if the equation or system of equations had been given to them?
- What do students need to think about in order to set up equations? What do they have to understand about equality? What do they have to understand about solution strategies to pick an appropriate set of facts for equations? How do these two sets of ideas need to be combined with the information given to get the equations?

Implications for Instruction:

Students need contexts when learning to combine and manipulate algebraic notation. If students merely spend time simplifying expressions in isolation of a context, then they will never develop a reason of why, when or how to uses algebraic notation in a problem situation. Students must have learning experiences around developing algebraic models in problem situations These must be non-routine situations and not a series of building algebraic models following a formalized process. Most students did not use algebra in this problem. That indicates a lack of familiarity with experiences where students need to develop and use algebraic models.

After a task like this has been given to students for informal assessment, the teacher might consider ways to design a lesson around the variety of strategies possible for solving the problem. The lesson might start by posing a hypothetical debate between two students. For example, "Cindy thinks that the problem can be solving by using the algebraic sum of the rows. Sam disagrees because the numerical value of the row is not given. Could Cindy be correct?" Then students could work in groups to see if they can use Cindy's idea. A different start might be, "Fred says that some of the information includes variables that cancel each other out. Would this be helpful? How would this help solve the problem?" Again students could work in groups and discuss ways to make this work. Consider, " Laura makes all the missing values into new variables. Kim says that the problem can be solved using only one variable. Juan says that his class has been working on solving two equations with two unknowns. Who is right? Could more than one person be right?" Having students examine a variety of strategies, helps them start to develop a generalization about classes of problems rather than solving each problem as a unique event. Just as we want students to develop generalizations for a function pattern, they need to be looking for generalizations about classes of problems. The discussion and debate process helps them make this transition. After students have found a variety of strategies, the teacher might ask further probing questions: "What did all these solutions have in common? How were they different? What did we have to know to use each one of the strategies? How were different pieces of information needed and unique to each solution type?" These types of questions help students reflect beyond the answer to making generalizations about what is needed to set up equations for solving problems.

This lesson structure is good for a variety of problems. Ideally, enough variety of solutions will come from the students in the class to provide the prompts for the lesson. However, the teacher should anticipate a variety of solution paths and be able to pull in some other strategies to push students' thinking if the students do not come up with them on their own. The prompts try not to lead or scaffold the student thinking, but give enough hints for students to do some thinking on their own and lead to debate. In the debating process, students are then confronted with ideas about justification. What makes a convincing argument? What does not make a convincing argument? In this process, students are also developing their logic as well as their problem-solving skills. Students are applying algebraic techniques and practicing procedures, but they are also learning the mathematics and big ideas behind the procedures and symbolic manipulation.

Teacher Notes:

Performance Assessment Task

Magic Squares
Grade 9

The task challenges a student to demonstrate understanding of the concepts representing and analyzing mathematical situations and structures with algebraic symbols. A student must understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. A student must determine equivalent forms of equations, inequalities, and systems of equations and solve them. A student must be able to employ forms of mathematical reasoning and proof appropriate to the solution of the problem using Magic Squares.

Common Core State Standards Math - Content Standards

High School – Algebra – Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Common Core State Standards Math – Standards of Mathematical Practice

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1), (x–1)(x2+x+1), and (x–1)(x3 +x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2005	8	5	39 %